

**UG 2<sup>nd</sup> Semester Examination - 2025 (Under NCCF)****Award : B.Sc****Discipline : Mathematics****Course Type : MDC 2****Course Code : MDC 201****Course Name : Mathematical Science****Full Marks - 35****Time - 2Hours****1. Answer any five questions :****1×5=5**

- Find the conjugate of  $\frac{1}{(1-i)^2}$ .
- If  $z = \bar{z}$  then z is purely imaginary. Justify the statement.
- If  $\vec{a} = (-2\hat{i} + 3\hat{j} + 5\hat{k})$ ,  $\vec{b} = (\hat{i} + 2\hat{j} + 3\hat{k})$ ,  $\vec{c} = (7\hat{i} - \hat{k})$  then show that the vectors are collinear.
- Which conic section is formed when  $\Delta \neq 0$  and  $D = ab - h^2 = 0$  in the general second-degree equation  $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$  ?
- In how many ways can 6 boys form a ring?
- Find the order and degree of the differential equation  $\frac{d^2y}{dx^2} = \left(1 + \frac{dy}{dx}\right)^{\frac{5}{2}}$ .
- Two coins are tossed at a time. Find the probability of getting one head and one tail.
- Find the unit vector parallel to the resultant of two vectors  $(2\hat{i} + 5\hat{j} - \hat{k})$  and  $(\hat{i} - \hat{j} - \hat{k})$ .

**2. Answer any five questions.****2×5=10**

- Compute the term independent of x in the expansion of  $\left(x^2 + \frac{1}{x}\right)^9$ .
- Find the angle between the two vectors  $\vec{a} = (6\hat{i} + 2\hat{j} + 3\hat{k})$  and  $\vec{b} = (2\hat{i} - 9\hat{j} + 6\hat{k})$ .
- Two unbiased dice are thrown simultaneously. Find the probability that the sum of the numbers on the faces is 8.
- If the events A and B are independent and  $P(A)=2/3$ ,  $P(B)=1/3$  then find  $P(A+B)$ .
- If  $|\vec{a}|=3$  and  $|\vec{b}|=4$  then find the values of the scalar c for which the vectors  $\vec{a} + c\vec{b}$  and  $\vec{a} - c\vec{b}$  are perpendicular to each other.
- Prove that  $\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}$ .
- Determine the differential equation whose primitive is  $y = (a + bx)e^{3x}$ , where a, b are arbitrary constants.

- h) A parabola has the equation  $(y - k)^2 = 4a(x - h)$ . If the focus of the parabola is at  $(3, 2)$  and the directrix is the line  $x = -1$ , what are values of  $h$ ,  $k$  and  $a$ ?

3. **Answer any two questions.**

**5×2=10**

- a) Find the eccentricity, foci and directrices of ellipse  $9x^2 + 25y^2 = 225$ . 1+2+2
- b) Show that the set of vectors  $(2\hat{i} + 3\hat{j} - \hat{k}), (-4\hat{i} + 2\hat{j} - 6\hat{k}), (5\hat{i} - 4\hat{j} + 9\hat{k})$  are linearly dependent.
- c) i) How many different factors can 2160 have?  
 ii) How many words can be made using all the letters in the word **MONDAY**? How many of them begin with **M** and do not end with **Y**? 2+3
- d) Solve:  $(x^2y - 2xy^2)dx + (3x^2y - x^3)dy = 0$ .

4. **Answer any one question.**

**10×1=10**

- a) i) If  $\vec{\alpha} = (\hat{i} + \hat{j} - 2\hat{k}), \vec{\beta} = (-\hat{i} + 2\hat{j} + 3\hat{k}), \vec{\gamma} = (5\hat{i} + 8\hat{k})$  then find  $c$  and  $d$  such that  $(\vec{\gamma} - c\vec{\alpha} - d\vec{\beta})$  is perpendicular to both  $\vec{\alpha}$  and  $\vec{\beta}$ .  
 ii) Find the equation to the circle which passes through the points  $(3, 4)$  and  $(3, -6)$  and which has its centre on the straight line  $2x + 3y = 3$ . 5+5
- b) i) Show that the product of all values of  $(1 + \sqrt{3}i)^{3/4}$  is 8.  
 ii) Find the value of  $k$  such that

$$f(x) = \begin{cases} kx(1-x); & \text{if } 0 < x \leq 1 \\ 0 & ; \text{otherwise} \end{cases}$$

is a possible probability density function. Also compute  $P\left(x > \frac{1}{2}\right)$ . 5+(2+3)

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