UG 2nd Semester Examination - 2025 (Under NCCF)

Award : B.Sc

Discipline : Mathematics

Course Type : MNC-2

Course Code : BSCMTMMN201

Course Name : Linear Algebra-I, Ordinary Differential Equations & Vector Calculus

Full Marks - 70 **Time - 3Hours**

1. Answer any five questions : **1×5=5**

a) Let $A=\begin{pmatrix} 1 & 3 \\ 4 & 5 \end{pmatrix}$, find the eigen values of A^3.

b) Find α such that the differential equation $\left(\frac{1}{x}+x^{\alpha}y^4\right)dx = y^3x^{\alpha+1}dy$ is exact.

c) Solve the differential equation $p^2+2xp-3x^2=0,\ p\equiv\frac{dy}{dx}$.

d) Find the solution of the differential equation : $(D^4+D^3-4D^2-4D)y=0,\ D\equiv\frac{d}{dx}$.

e) If $\overrightarrow{f(t)}$ has a constant magnitude, show that $\overrightarrow{f(t)}$ is perpendicular to $\frac{d\overrightarrow{f(t)}}{dt}$.

f) When is a matrix said to be singular? Give an example.

g) Give an example of skew symmetric matrix.

h) Find the equation of the tangent plane to the surface xyz =4 at the point (1,2,2).

2. Answer any ten questions. **2×10=20**

a) Verify Cayley Hamilton Theorem for the matrix $A=\begin{pmatrix} 1 & 2 \\ -2 & 3 \end{pmatrix}$.

b) Show that the system of equations $x+2y-z=5$; $3x-y+2z=3$; $ax+y+z=2a$ have a unique solution if $a\neq 4$.

c) Reduce the equation $y=2px+y^2p^3$ to Clairaut's form.

d) Find the solution of the initial value problem, $y\frac{d^2y}{dx^2}+3\left(\frac{dy}{dx}\right)^2=0$, $y(0)=1$, $\left(\frac{dy}{dx}\right)_{x=0}=6$.

e) Find the Wronskian of a pair of linearly independent solutions of $\frac{d^2y}{dx^2}+4y=0$.

h) Solve ; $(D^2-4)y=e^{2x}+3x$, where $D\equiv\frac{d}{dx}$.

g) Solve : $\sin x\frac{dy}{dx}-y\cos x+y^2=0$.

h) Find the curve which satisfies the equation $\frac{dy}{dx}=\frac{1+y^2}{xy}$ and passes through (1,0).

i) Find the volume of the parallelopiped whose edges are $\vec{a}=2\hat{i}-4\hat{j}+5\hat{k}$, $\vec{b}=\hat{i}-\hat{j}+\hat{k}$ and $\vec{c}=3\hat{i}-5\hat{j}+2\hat{k}$.

j) If $\vec{\alpha} = (2y+3)\hat{i} + xz\hat{j} + (yz - x)\hat{k}$, evaluate $\int \vec{\alpha} \cdot d\vec{r}$ over C, from $t=0$ to $t=1$ Where C is the curve $x = 2t^2,\ y = t,\ z = t^3$.

k) Express $A = \begin{bmatrix} 4 & 5 & 1 \\ 3 & 7 & 2 \\ 1 & 6 & 8 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix.

l) Let $f = x^3 + y^3 + z^3$, then find the directional derivative of f at (1,–1, 2) in the direction of the vector $\hat{i} + 2\hat{j} + \hat{k}$.

m) If $\vec{r} = 3t\,\hat{i} + 3t^2\hat{j} + 2t^3\hat{k}$ find the value of $\left[\frac{dr}{dt}, \frac{d^2r}{dt^2}, \frac{d^3r}{dt^3}\right]$.

n) Find unit normal to the surface $-x^2yz^2 + 2xy^2z = 0$ at the point $P(1,1,1)$.

3. Answer any five questions : **5×5 = 25**

a) Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 4 & -4 & 2 \\ 2 & -2 & 2 \\ 0 & 0 & 1 \end{bmatrix}$.

b) Apply the method of variation of parameters to solve $\frac{d^3y}{dx^3} - y = \frac{2}{1+e^x}$.

c) Solve the total differential equation $z^2dx + (z^2 - 2yz)dy + (2y^2 - yz - zx)dz = 0$.

Solve : $\frac{dy}{dx} + \frac{y}{x}\log_e y = \frac{y}{x^2}(\log_e y)^2$.

d) Solve : $\frac{dx}{dt} - 7x + y = 0;\ \frac{dy}{dt} - 2x - 5y = 0$.

e) Find the vector equation of a plane containing two parallel straight lines $\vec{r} = \vec{a} + t\vec{c}$ and $\vec{r} = \vec{b} + s\vec{c}$ (t and s are scalars).

f) Show that the matrix $\begin{bmatrix} 2 & 0 & 1 \\ 3 & 3 & 0 \\ 6 & 2 & 3 \end{bmatrix}$ is non- singular, hence express it as a product of elementary matrices.

g) Show that the general solution of the differential equation $\frac{dy}{dx} + py = Q$ can be written in the form

$y = \frac{Q}{p} - e^{-\int Pdx}\left[\int e^{\int Pdx} d\left(\frac{Q}{P}\right) + C\right]$, where P and Q are functions of x and C is an arbitrary constant.

h) Solve $(D^2 + a^2)\,y = \sec\,(a\,x)$, $D \equiv \frac{d}{dx}$.

4. Answer any two questions. **10×2=20**

a) i) Find the row -reduced Echelon form of the matrix $\begin{bmatrix} 2 & 0 & 4 & 2 \\ 3 & 2 & 6 & 5 \\ 5 & 2 & 10 & 7 \\ 0 & 3 & 2 & 5 \end{bmatrix}$.

ii) Solve : $\dfrac{dx}{z(x+y)} = \dfrac{dy}{z(x-y)} = \dfrac{dz}{x^2 - y^2}$. 5+5

b) i) Solve the differential equation $\dfrac{d^2 y}{dx^2} - \dfrac{1}{x}\dfrac{dy}{dx} - 3x^2 y = 4x^2 \sin(x^2)$.

ii) Show that div $(\vec{E} \ X \ \vec{F}) = \vec{F}.\ curl\,\vec{E} - \vec{E}.curl\,\vec{F}$. 5+5

c) i) Find the algebraic and geometric multiplicities of each eigen value of the matrix

$$\begin{bmatrix} 1 & -1 & 0 \\ 1 & 2 & -1 \\ 3 & 2 & -2 \end{bmatrix}$$

ii) In the differential equation $Mdx + Ndy = 0,\ if\ Mx + Ny \neq 0$ and the equation is homogeneous then show that $\dfrac{1}{Mx + Ny}$ is an integrating factor of the equation.

Hence solve : $(x^2 y - 2xy^2)dx - (x^3 - 3x^2 y)dy = 0$. 4+2+4

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